Actual vs expected return for investing strategies based on the Markowitz model

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MPaR'21,22-23 March 2021

keywords: Markowitz model, portfolio selection, risk-return profile, investing strategy

1. Markowitz model and multi-period models
2. Decision-maker's risk-return profiles
3. Multi-period investment with the Markowitz model
4. Experiment design
5. Results

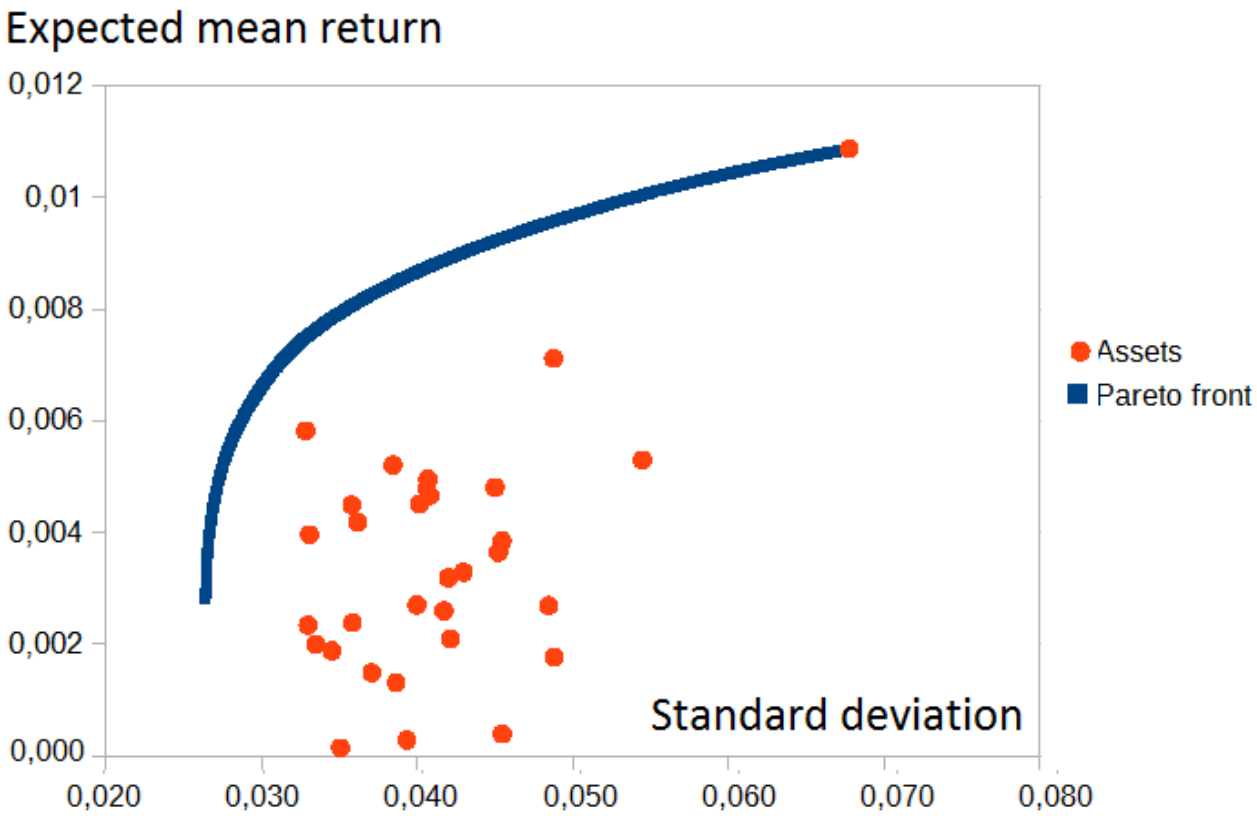
A single-period (static) Markowitz mean–standard deviation model*

$$\begin{aligned} &\min\ f_1(x) = \sqrt{x^T Q x} \quad \text{(minimize standard deviation)} \\ &\max\ f_2(x) = e^T x \qquad\qquad \text{(maximize expected mean)} \end{aligned} \tag{1}$$

$$\text{subject to } x \in X_0 = \left\{ x \;\middle|\; \begin{array}{l} u^T x = 1, \text{ (all capital to be consumed)}, \\ \\ x \geqslant 0, \end{array} \right\},$$

where x is the vector of fractions of the capital spent on buying individual assets, Q is the covariance matrix, e is the vector of expected means, u is the all-ones vector.

* Markowitz H., Portfolio selection – efficient diversification of investments, John Wiley and Sons Inc., New York, 1959.



Pareto front in the Markowitz model as it is often seen in the literature (the case of Beasley OR–library set with 31 assets)

Multi-period models in the expected mean–standard deviation framework capture

- Intertemporal effects (a discrete time economy);
- Market impact on asset prices from buying and selling assets;
- Market impact costs associated with the immediacy of trading (a trade-off between the cost of the immediacy of trading AND price volatility).

A critique of multi-period models

- It is often difficult to accurately estimate return/risk for multiple periods;
- It is often difficult to accurately estimate market impact effects;
- These models are computationally intensive (for a large number of assets).

Kolm P., Tütüncü R., Fabozzi F. "60 Years of portfolio optimization: Practical challenges and current trends", European Journal of Operational Research, Vol. 234, Is. 2, pp. 356-371, 2014

"For these reasons, practitioners typically use single-period models to rebalance the portfolio from one period to another."

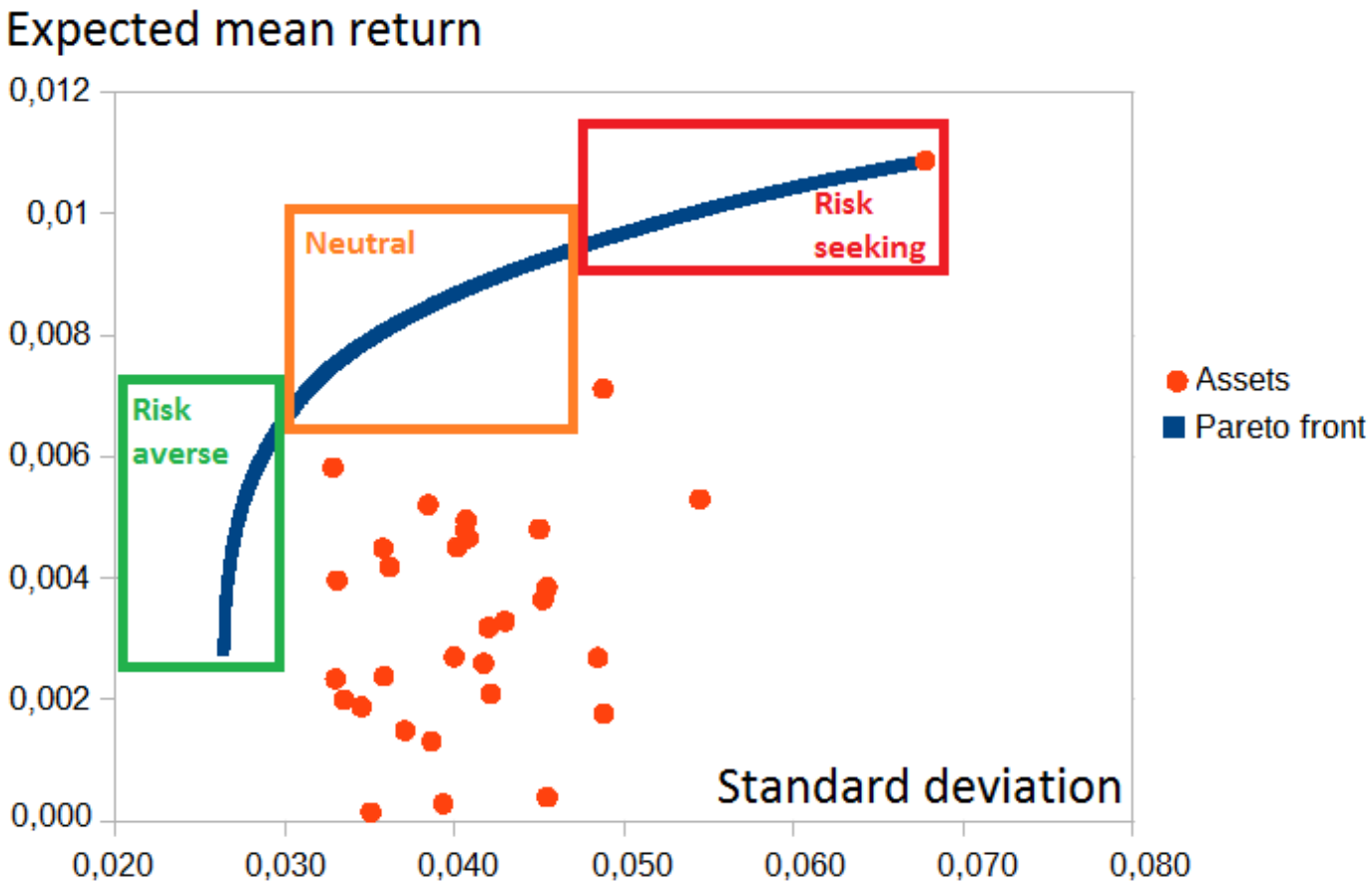
Motivation and research question

1. What is the efficiency of the popular investing strategies (e.g. buy-and-hold, one-month rebalancing) in the dynamic environment with the Markowitz model as a base model?
2. What is the impact of the risk level on the actual (realized) return?
3. Are calendar-based rebalancing strategies better than the (passive) multi-period B&H strategy?

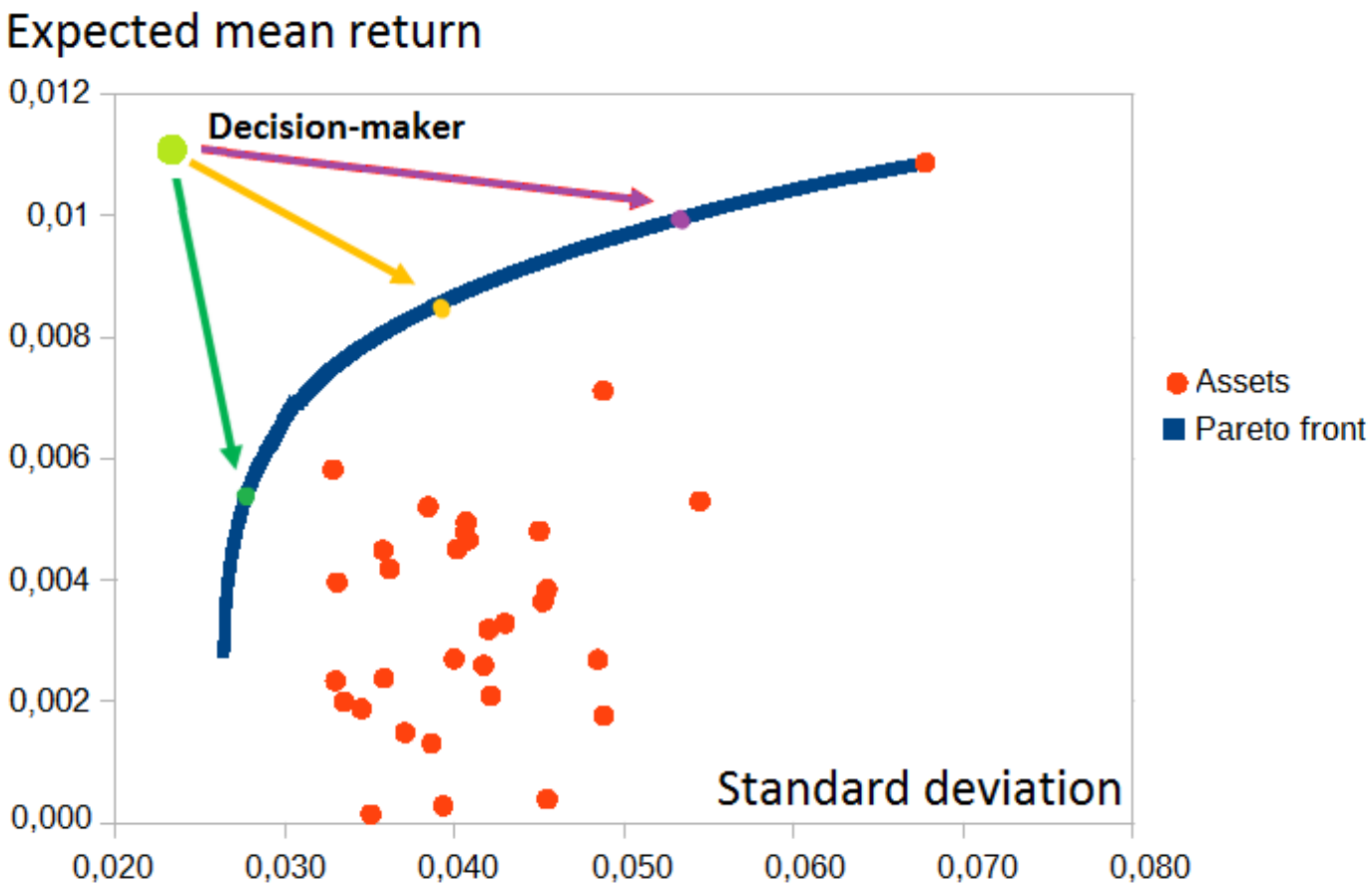
Multi-period models in literature

- Calvet L., Campbell J., Sodini P. "Fight or Flight? Portfolio Rebalancing by Individual Investors", The Quarterly Journal of Economics, Vol. 124, Is. 1, pp. 301-348, 2009.
- Dichtl H., Drobetz W., Wambach M. "Where is the value added of rebalancing? A systematic comparison of alternative rebalancing strategies", Financial Markets and Portfolio Management, Vol. 28, pp. 209-231, 2014.
- Guastaroba G., Mansini R., Grazia Speranza M., "Models and Simulations for Portfolio Rebalancing", Computational Economics, 33, 2009.
- Hilliard J.E., Hilliard J. "Rebalancing versus buy and hold: theory, simulation and empirical analysis". Review of Quantitative Finance and Accounting, 50(1), 2018.

The decision maker's preferences in portfolio reconstruction are not examined in these works.



Pareto front for the example problem with 31 assets. Three different risk-return segments of the Pareto front.



Pareto front for the example problem with 31 assets. Three different risk-return profiles (the DM's preferences).

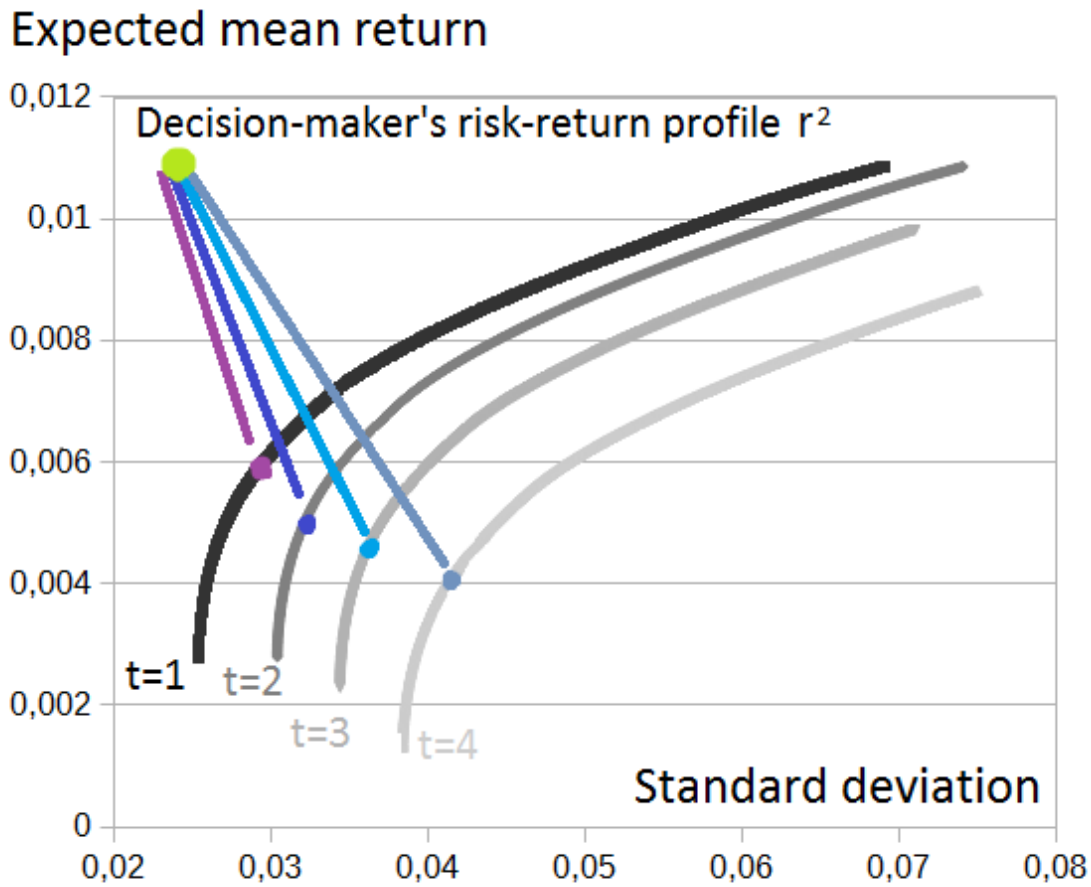
- investing year $y \in \{2014, 2015, 2016, 2017\}$;
- for every investing year, a set of instruments A^n, with $n \in \{200, 300, ..., 600\}$;
- investing interval for year y covers 13 months: $m_{y,1}$ - January for the investing year y; $m_{y,2}$ - February for the investing year y; $m_{y,13} = m_{y+1,1}$;
- covariance matrix $S_{y,t}(A^n)$ along with the vector of returns $\mu_{y,t}(A^n)$ is defined for months $m_{y,1}$ to $m_{y,12}$.

Short-term investing

- to derive an efficient portfolio with minimal variance we define problem $\prod(S_{y,t}(A^n), \mu_{y,t}(A^n))$ – variance minimization s.t. constraints in the Markowitz model;
- to derive an efficient portfolio for an expected return r we define a problem $\prod(S_{y,t}(A^n), \mu_{y,t}(A^n), r)$ – variance minimization s.t. constraints in the Markowitz model AND $\mu_{y,t}(A^n) = r$.

- r^{min} – expected return of the solution to $\prod(S_{y,t}(A^n), \mu_{y,t}(A^n))$; r^{max} – maximal value of the expected return on the Pareto front;
- m, $m > 0$, is the number of risk-return profiles;
- divide $[r^{min}, r^{max}]$ into m equal-sized intervals (risk-return segments) $I^k, k = 1, ..., m$;
- r^k is the middle of interval I^k; r^k determines the k'th risk-return profile; r^1 is the least risky profile, and r^m – the most risky one.

By solving $\prod(S_{y,t}(A^n), \mu_{y,t}(A^n), r^k)$ the efficient portfolio corresponding to the k'th risk-return profile is derived



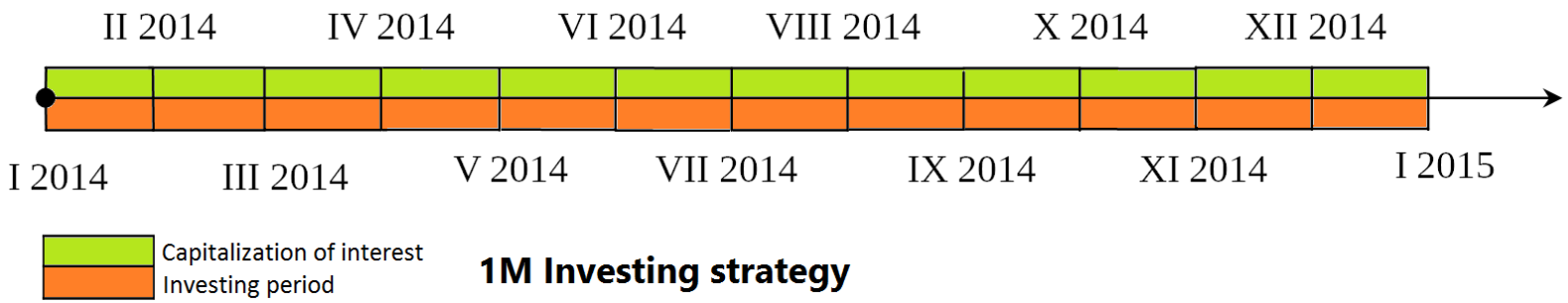
Dynamic investment: choosing an efficient portfolio, corresponding to a risk-return profile, in $t = 1, 2, 3, 4$.

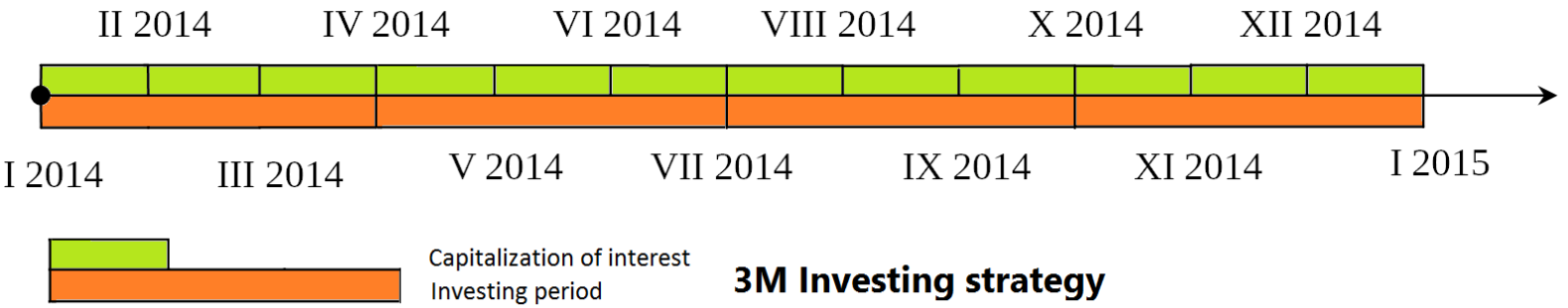
Multi-period investing strategies

- M1 investing strategy - portfolio change every month;
- M3 investing strategy - portfolio change every 3 months;
- M6 investing strategy - portfolio change every 6 months;
- Multi-period B&H investing strategy - portfolio unchanged during the whole investing interval;

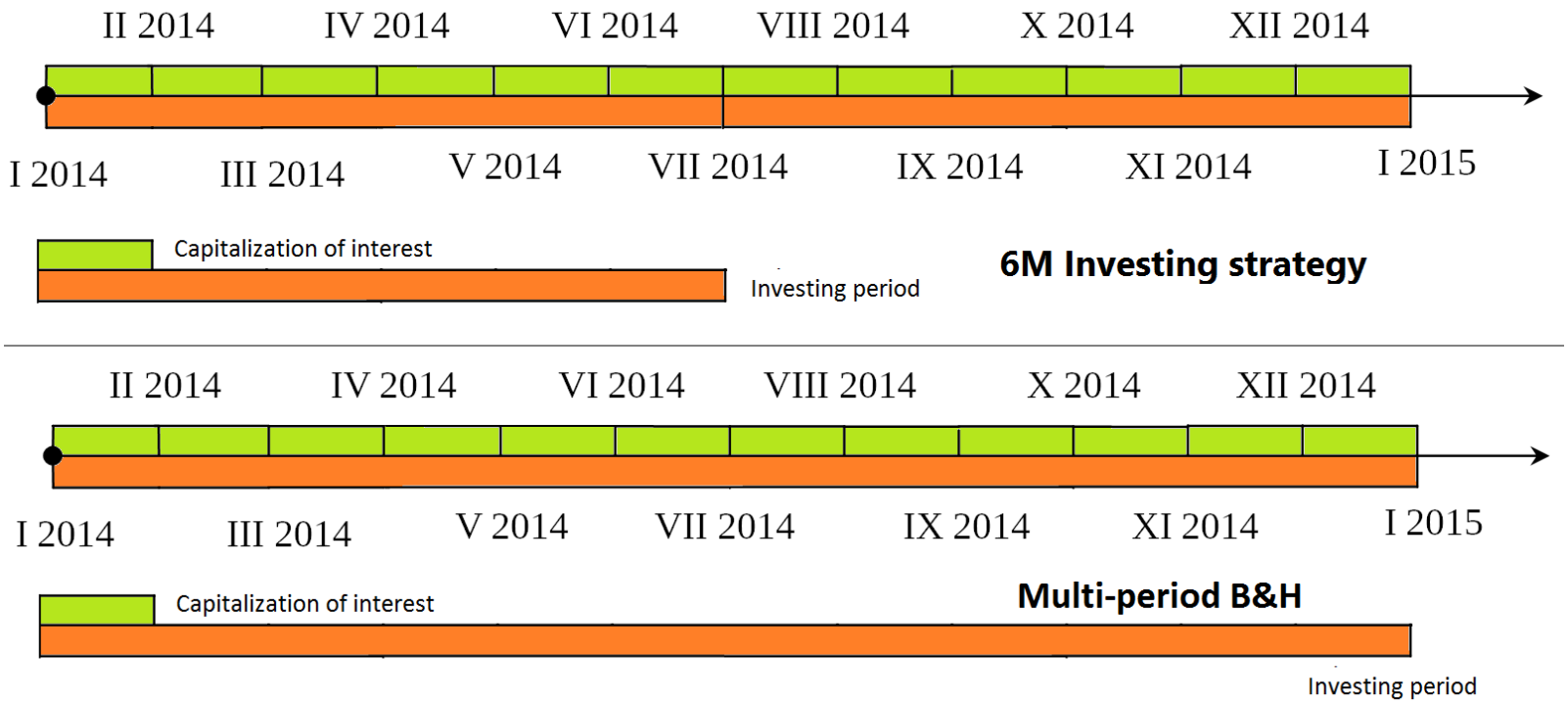
M1, M3, M6 – calendar-based rebalancing.

Capitalization of interest done every month (for every investing strategy)

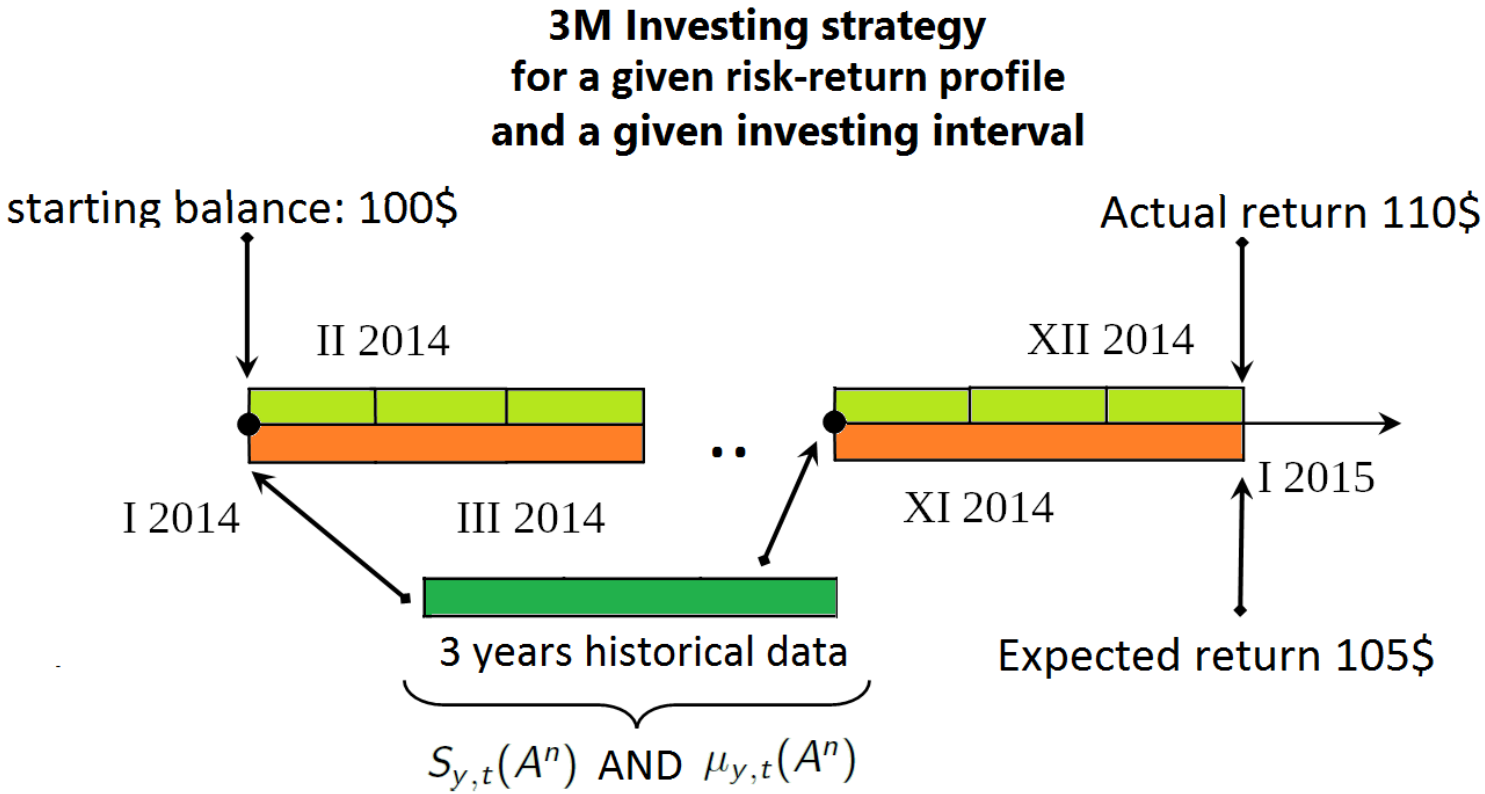




Comparison of M1 and M3 investing strategies.



Comparison of M6 and Multi-period B&H investing strategies.



Solving 4 $\prod$ problems (portfolio rebalancing) based on the 3 years historical data.

Experiment data

- 5 datasets;
- the single dataset includes 200 to 600 instruments;
- every dataset divided into 4 investing intervals covering 13 successive months;
- 5 different risk-return profiles;
- 4 different investing strategies:
  - M1 – covering 48 investing periods;
  - M3 – covering 16 investing periods;
  - M6 – covering 8 investing periods;
  - Multi-period B&H covering 4 investing periods.
- we assume, that 100$ is invested at the beginning of the investing interval.

y_1	Increasing risk →				
	r^1	r^2	r^3	r^4	r^5
1M	-0,024	0,090	0,163	0,248	0,375
3M	-0,111	0,054	0,181	0,322	0,510
6M	-0,323	-0,148	-0,009	0,145	0,346
Multi-period B&H	0,002	0,029	0,058	0,096	0,114
1M	-0,001	0,170	0,254	0,422	0,521
3M	-0,023	0,186	0,310	0,521	0,664
6M	-0,162	0,043	0,163	0,369	0,509
Multi-period B&H	0,001	0,033	0,070	0,122	0,166
1M	-0,024	0,124	0,192	0,290	0,227
3M	-0,129	-0,025	-0,003	0,046	-0,070
6M	-0,307	-0,260	-0,299	-0,314	-0,498
Multi-period B&H	-0,002	0,026	0,065	0,130	0,167
1M	-0,024	0,119	0,198	0,292	0,228
3M	-0,110	-0,015	0,014	0,054	-0,066
6M	-0,293	-0,255	-0,285	-0,308	-0,495
Multi-period B&H	0,008	0,029	0,063	0,129	0,168
1M	-0,117	0,397	1,010	1,488	1,877
3M	-0,173	0,154	0,568	0,837	1,003
6M	-0,405	-0,324	-0,169	-0,174	-0,296
Multi-period B&H	-0,042	0,166	0,430	0,639	0,845

Differences between the expected mean return (predicted by the Markowitz model) and the actual return in **the first investing interval** (year 2014).

Definition

Reliability of the investing strategy = |exp. mean return - actual return| (precision accuracy)

y_1					
	r^1	r^2	r^3	r^4	r^5
200					
1M	0,024	0,090	0,163	0,248	0,375
3M	0,111	0,054	0,181	0,322	0,510
6M	0,323	0,148	0,009	0,145	0,346
Multi-period B&H	0,002	0,029	0,058	0,096	0,114
300					
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3M	0,023	0,186	0,310	0,521	0,664
6M	0,162	0,043	0,163	0,369	0,509
Multi-period B&H	0,001	0,033	0,070	0,122	0,166
400					
1M	0,024	0,124	0,192	0,290	0,227
3M	0,129	0,025	0,003	0,046	0,070
6M	0,307	0,260	0,299	0,314	0,498
Multi-period B&H	0,002	0,026	0,065	0,130	0,167
500					
1M	0,024	0,119	0,198	0,292	0,228
3M	0,110	0,015	0,014	0,054	0,066
6M	0,293	0,255	0,285	0,308	0,495
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600					
1M	0,117	0,397	1,010	1,488	1,877
3M	0,173	0,154	0,568	0,837	1,003
6M	0,405	0,324	0,169	0,174	0,296
Multi-period B&H	0,042	0,166	0,430	0,639	0,845

Values of reliability of the investing strategy and winning strategies in **the first investing interval** (year 2014).

y_2					
	r^1	r^2	r^3	r^4	r^5
			200		
1M	0,130	0,357	0,533	0,775	1,073
3M	0,294	0,626	0,914	1,273	1,694
6M	0,518	0,909	1,260	1,685	2,175
Multi-period B&H	0,024	0,061	0,101	0,149	0,135
			300		
1M	0,119	0,223	0,420	0,730	0,998
3M	0,324	0,524	0,822	1,239	1,618
6M	0,598	0,847	1,196	1,668	2,105
Multi-period B&H	0,031	0,044	0,071	0,128	0,134
			400		
1M	0,081	0,231	0,541	0,828	1,234
3M	0,384	0,840	1,477	2,113	2,892
6M	0,763	1,482	2,400	3,337	4,438
Multi-period B&H	0,020	0,050	0,115	0,165	0,207
			500		
1M	0,076	0,253	0,564	0,848	1,234
3M	0,366	0,852	1,492	2,128	2,890
6M	0,718	1,472	2,399	3,342	4,433
Multi-period B&H	0,020	0,054	0,118	0,165	0,207
			600		
1M	0,053	0,252	0,544	0,867	1,229
3M	0,272	0,793	1,429	2,120	2,875
6M	0,556	1,357	2,294	3,308	4,409
Multi-period B&H	0,011	0,053	0,119	0,164	0,206

Values of reliability of the investing strategy and winning strategies in **the second investing interval** (year 2015).

y_3					
	r^1	r^2	r^3	r^4	r^5
			200		
1M	0,077	0,183	0,364	0,801	1,410
3M	0,297	0,674	1,135	1,861	2,769
6M	0,443	1,078	1,808	2,813	4,010
Multi-period B&H	0,032	0,053	0,113	0,258	0,436
			300		
1M	0,153	0,267	0,463	0,816	1,413
3M	0,400	0,769	1,230	1,857	2,736
6M	0,535	1,019	1,599	2,348	3,355
Multi-period B&H	0,049	0,070	0,120	0,227	0,344
			400		
1M	0,151	0,280	0,497	0,883	1,429
3M	0,327	0,715	1,200	1,864	2,696
6M	0,339	0,867	1,497	2,312	3,301
Multi-period B&H	0,048	0,078	0,135	0,239	0,347
			500		
1M	0,133	0,269	0,485	0,866	1,429
3M	0,347	0,734	1,210	1,859	2,700
6M	0,416	0,930	1,540	2,327	3,311
Multi-period B&H	0,053	0,078	0,135	0,238	0,347
			600		
1M	0,090	0,264	0,475	0,853	1,424
3M	0,224	0,665	1,154	1,818	2,686
6M	0,257	0,833	1,462	2,273	3,292
Multi-period B&H	0,035	0,076	0,133	0,235	0,345

Values of reliability of the investing strategy and winning strategies in **the third investing interval** (year 2016).

y_4					
	r^1	r^2	r^3	r^4	r^5
			200		
1M	0,006	0,027	0,076	0,092	0,110
3M	0,043	0,018	0,022	0,029	0,038
6M	0,065	0,156	0,264	0,340	0,420
Multi-period B&H	0,002	0,010	0,017	0,018	0,026
			300		
1M	0,018	0,033	0,088	0,105	0,159
3M	0,045	0,005	0,039	0,045	0,088
6M	0,030	0,096	0,227	0,324	0,459
Multi-period B&H	0,013	0,002	0,013	0,013	0,025
			400		
1M	0,039	0,014	0,071	0,093	0,146
3M	0,144	0,111	0,075	0,075	0,045
6M	0,094	0,001	0,097	0,161	0,258
Multi-period B&H	0,012	0,002	0,010	0,012	0,025
			500		
1M	0,055	0,000	0,014	0,107	0,151
3M	0,172	0,134	0,139	0,066	0,041
6M	0,094	0,002	0,049	0,180	0,264
Multi-period B&H	0,010	0,002	0,006	0,003	0,027
			600		
1M	0,075	0,026	0,033	0,166	0,225
3M	0,165	0,134	0,093	0,020	0,061
6M	0,150	0,039	0,084	0,282	0,409
Multi-period B&H	0,015	0,006	0,004	0,008	0,029

Values of reliability of the investing strategy and winning strategies in **the fourth investing interval** (year 2017).

Average over all risk-return profiles

γ_1	1M	3M	6M	Multi-period B&H
200	1,800	2,356	1,941	0,599
300	2,736	3,407	2,491	0,784
400	1,715	0,546	3,356	0,780
500	1,720	0,518	3,271	0,794
600	9,779	5,470	2,735	4,246

Average over all risk-return profiles

γ_2	1M	3M	6M	Multi-period B&H
200	5,737	9,605	13,094	0,940
300	4,981	9,054	12,825	0,814
400	5,829	15,410	24,838	1,113
500	5,951	15,458	24,727	1,129
600	5,890	14,979	23,847	1,105

Average over all risk-return profiles

γ_3	1M	3M	6M	Multi-period B&H
200	5,673	13,470	20,305	1,785
300	6,224	13,984	17,713	1,618
400	6,480	13,604	16,629	1,695
500	6,363	13,700	17,048	1,703
600	6,212	13,095	16,234	1,648

Average over all risk-return profiles

γ_4	1M	3M	6M	Multi-period B&H
200	0,625	0,302	2,491	0,146
300	0,804	0,444	2,270	0,132
400	0,727	0,900	1,221	0,120
500	0,655	1,105	1,178	0,095
600	1,050	0,947	1,927	0,124

Average values of reliability of the investing strategy over all risk-return profiles.

y_1					
	r^1	r^2	r^3	r^4	r^5
			200		
1M	0,024	0,090	0,090	0,163	0,163
3M	0,111	0,054	0,054	0,181	0,181
6M	0,323	0,148	0,148	0,009	0,009
Multi-period B&H	0,002	0,029	0,058	0,096	0,114
			300		
1M	0,001	0,170	0,254	0,422	0,521
3M	0,023	0,186	0,310	0,521	0,664
6M	0,162	0,043	0,163	0,369	0,509
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Multi-period B&H	0,002	0,026	0,065	0,130	0,167
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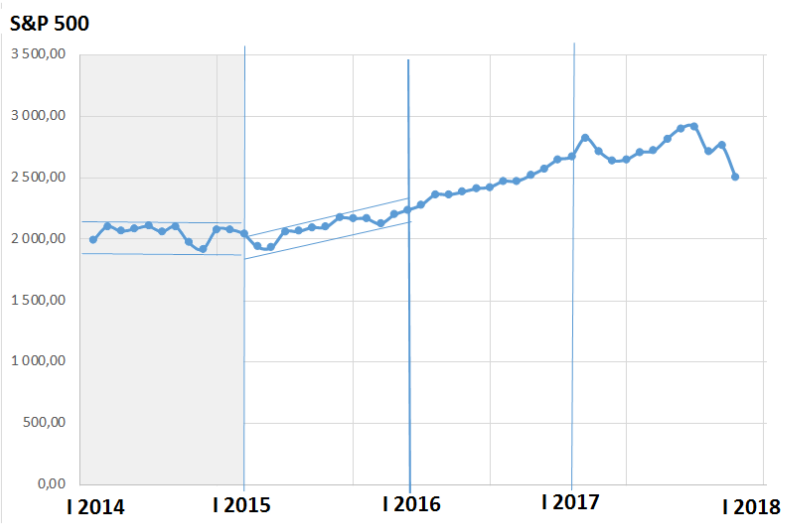
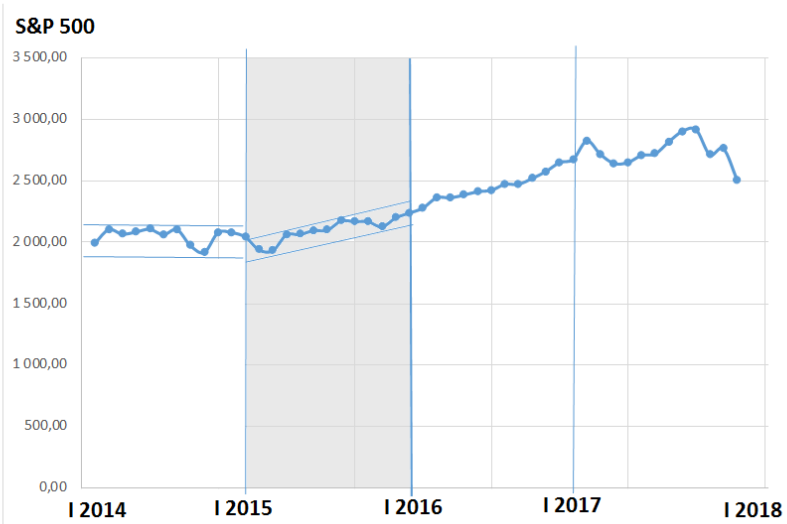
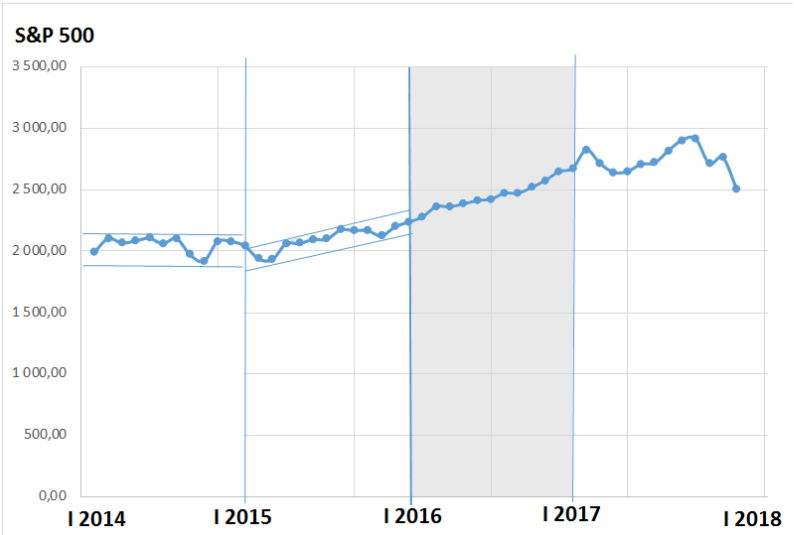


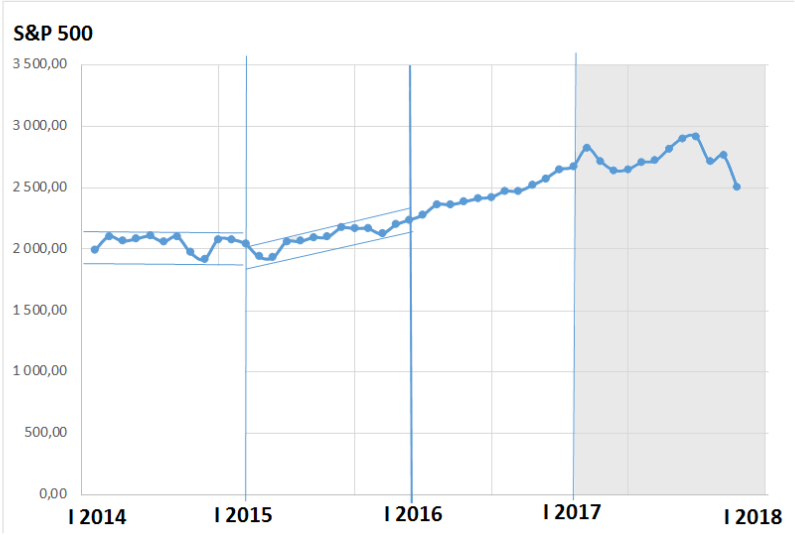


Comparision of the results with the S&P 500 market index – for the first two investing intervals.

y_3					
	r^1	r^2	r^3	r^4	r^5
			200		
1M	0,077	0,183	0,364	0,801	1,410
3M	0,297	0,674	1,135	1,861	2,769
6M	0,443	1,078	1,808	2,813	4,010
Multi-period B&H	0,032	0,053	0,113	0,258	0,436
			300		
1M	0,153	0,267	0,463	0,816	1,413
3M	0,400	0,769	1,230	1,857	2,736
6M	0,535	1,019	1,599	2,348	3,355
Multi-period B&H	0,049	0,070	0,120	0,227	0,344
			400		
1M	0,151	0,280	0,497	0,883	1,429
3M	0,327	0,715	1,200	1,864	2,696
6M	0,339	0,867	1,497	2,312	3,301
Multi-period B&H	0,048	0,078	0,135	0,239	0,347
			500		
1M	0,133	0,269	0,485	0,866	1,429
3M	0,347	0,734	1,210	1,859	2,700
6M	0,416	0,930	1,540	2,327	3,311
Multi-period B&H	0,053	0,078	0,135	0,238	0,347
			600		
1M	0,090	0,264	0,475	0,853	1,424
3M	0,224	0,665	1,154	1,818	2,686
6M	0,257	0,833	1,462	2,273	3,292
Multi-period B&H	0,035	0,076	0,133	0,235	0,345

y_4					
	r^1	r^2	r^3	r^4	r^5
			200		
1M	0,006	0,027	0,076	0,092	0,110
3M	0,043	0,018	0,022	0,029	0,038
6M	0,065	0,156	0,264	0,340	0,420
Multi-period B&H	0,002	0,010	0,017	0,018	0,026
			300		
1M	0,018	0,033	0,088	0,105	0,159
3M	0,045	0,005	0,039	0,045	0,088
6M	0,030	0,096	0,227	0,324	0,459
Multi-period B&H	0,013	0,002	0,013	0,013	0,025
			400		
1M	0,039	0,014	0,071	0,093	0,146
3M	0,144	0,111	0,075	0,075	0,045
6M	0,094	0,001	0,097	0,161	0,258
Multi-period B&H	0,012	0,002	0,010	0,012	0,025
			500		
1M	0,055	0,000	0,014	0,107	0,151
3M	0,172	0,134	0,139	0,066	0,041
6M	0,094	0,002	0,049	0,180	0,264
Multi-period B&H	0,010	0,002	0,006	0,003	0,027
			600		
1M	0,075	0,026	0,033	0,166	0,225
3M	0,165	0,134	0,093	0,020	0,061
6M	0,150	0,039	0,084	0,282	0,409
Multi-period B&H	0,015	0,006	0,004	0,008	0,029





Comparision of the results with the S&P 500 market index – for the last two investing intervals.

Conclusion

- Multi-period B&H strategy for most cases seems to be superior over rebalancing strategies (in a sense of reliability of the investing strategy).
- Calculations for the more complex models (than the Markowitz model) and derivation of the 2280 efficient portfolios could be far more computationally demanding.

Open questions / future works

- What is the impact of the market index on the presented results?
- New data and the long-term investment covering approx. 10 years;
- Other expected return–variance portfolio selection models could be considered in the presented framework (e.g., Markowitz model with cardinality constraints).

Thank you for your attention